

9

STATISTICS AND PROBABILITY

PROBABILITY

Probability plays an important part in our daily lives. The study of chance has applications in weather forecasting, predicting disease, political campaigns, insurance and gambling. Insurance companies use probability to assess the risk of an event occurring. Statistics show that the age and sex of a driver affects the chances of a car accident, and the more distance a person drives, the higher the risk of an accident. These statistics determine how much drivers pay to insure their cars.



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Chapter outline

	Working mathematically				
9.01 Probability	U	F		R	C
9.02 Complementary events	U	F	PS	R	C
9.03 Venn diagrams	U	F	PS	R	C
9.04 Two-way tables	U	F		R	C
9.05 Probability problems	U	F	PS	R	C
9.06 Relative frequency	U	F		R	C

Wordbank

at least Referring to the smallest number, for example, 'at least 2' means 2, 3, 4, ..., that is, '2 or more'

complementary event The 'opposite' event (for example, the complementary event to selecting an Ace from a deck of cards is *not* selecting an Ace)

expected frequency The expected number of times an event will occur over repeated trials

mutually exclusive events Events or categories that have no items in common

trial One go or run of a repeated probability experiment, for example, one roll of a die

two-way table A table that shows the number of items belonging to overlapping categories

Venn diagram A diagram that uses circles (usually overlapping) to group items into categories



Probability

In this chapter you will:

- use the sample space to calculate the probability of an event
- solve problems involving complementary events
- use Venn diagrams and two-way tables to represent sample spaces and events to solve probability problems
- identify 2 or more events that are mutually exclusive or overlapping
- describe events using such as 'and', exclusive 'or', inclusive 'or' and 'at least'
- compare the observed frequency of an event with its expected frequency

SkillCheck ANSWERS ON P. 572



The language of chance



Chance cards



Describing probability

1 Convert each number into a decimal.

a $\frac{17}{20}$

b $\frac{3}{8}$

c 2%

d 30%

2 List all the possible outcomes for each situation.

a tossing a coin

b rolling a die

c the gender of a baby

d the colour of traffic lights

e the result of a football game

f the result of a driving test

3 Convert each number into a percentage.

a 0.7

b 0.56

c $\frac{2}{3}$

d $\frac{6}{25}$

4 Evaluate each expression.

a $1 - \frac{1}{3}$

b $1 - \frac{2}{5}$

c $1 - \frac{3}{8}$

5 Which term best describes the chance that the next baby born in Australia is a girl? Select the correct answer **A**, **B**, **C** or **D**.

A certain

B definite

C even chance

D probable

6 Convert each number into a simplified fraction.

a 0.28

b 0.02

c 64%

d 80%

7 Is the chance of each event greater than or less than $\frac{1}{2}$?

a You being home by 4 p.m. this afternoon

b You winning Lotto one day

c You listening to the radio today

d You learning to drive next year

e You going interstate this year

f You sending a text message on your phone today

In probability (chance) situations, the set of all possible outcomes is called the **sample space**. For example, if a coin is tossed, the sample space is {heads, tails}. If each outcome has an equal chance, then we say that each outcome is **equally likely**.

An **event** consists of one or more outcomes. For example, the event that a baby is born in a summer month consists of the outcomes {December, January, February}. An event has the symbol E , and we can calculate its probability as a fraction.

Probability of an event

$P(E)$ means ‘the probability of an event, E (occurring)’.

If all possible outcomes are **equally likely**, then:

$$P(E) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$$

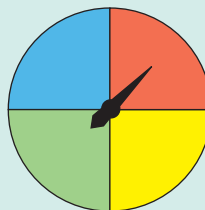
$$\text{or } P(E) = \frac{\text{number of outcomes matching } E}{\text{number of outcomes in the sample space}}$$

A **favourable outcome** is one of the outcomes in the event that you want, whose probability you are calculating.

The **probability** of an event can be written as a fraction, decimal or percentage. For example, the probability of tossing tails on a coin can be written as $\frac{1}{2}$, 0.5 or 50%.

Example 1

- Write the sample space for this spinner.
- Is each outcome equally likely?
- Find the probability that the spinner lands on red.
- Find the probability that the spinner lands on a ‘traffic light’ colour.



Solution

- The sample space is {red, yellow, green, blue}.
- Each coloured region is equal in size ($\frac{1}{4}$ of the circle), so each outcome is equally likely.
- $P(\text{red}) = \frac{1}{4}$
- $P(\text{traffic light colour}) = P(\text{red or yellow or green})$

$$= \frac{3}{4}$$

One chance in 4

3 favourable outcomes out of 4



Sample space



Basic probability



Greedy Pig game



Basic probability



Games of chance



Probability problems



Spinner game



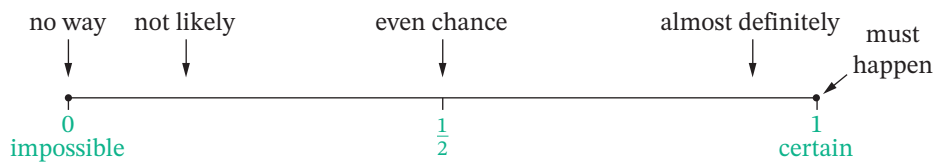
Theoretical probabilities

The language of probability

Probability term	In Example 1
An experiment (or chance experiment) is a situation involving chance that leads to results called outcomes.	Spinning a spinner
A trial is one go or run of the experiment.	One spin of the spinner
An outcome is the result of an experiment.	The arrow landing on red
The sample space is the set of all possible outcomes.	{red, yellow, green, blue}
An event is one or more outcomes of an experiment.	The arrow landing on a 'traffic light' colour: red, yellow or green
In a random experiment, every possible outcome has the same chance of occurring.	All spins on this spinner are random because every colour has the same chance.

The range of probability

Because probability is a fraction, its value must range from 0 to 1, or as a percentage, from 0% to 100%.



Example 2

A die is rolled. Find the probability that the number rolled is:

- a** divisible by 3
- b** a factor of 6
- c** less than 7 (answer as a percentage)
- d** **at least** 4 (answer as a decimal)

Solution

There are 6 possible outcomes when a die is rolled: {1, 2, 3, 4, 5, 6}

- a** $P(\text{divisible by } 3) = \frac{2}{6} = \frac{1}{3}$ 2 numbers {3 and 6}
- b** $P(\text{a factor of } 6) = \frac{4}{6} = \frac{2}{3}$ 4 numbers {1, 2, 3, 6}
- c** $P(\text{less than } 7) = \frac{6}{6} = 100\%$ All outcomes are less than 7: a certain event
- d** $P(\text{at least } 4) = \frac{3}{6} = \frac{1}{2} = 0.5$ 3 numbers {4, 5, 6}.

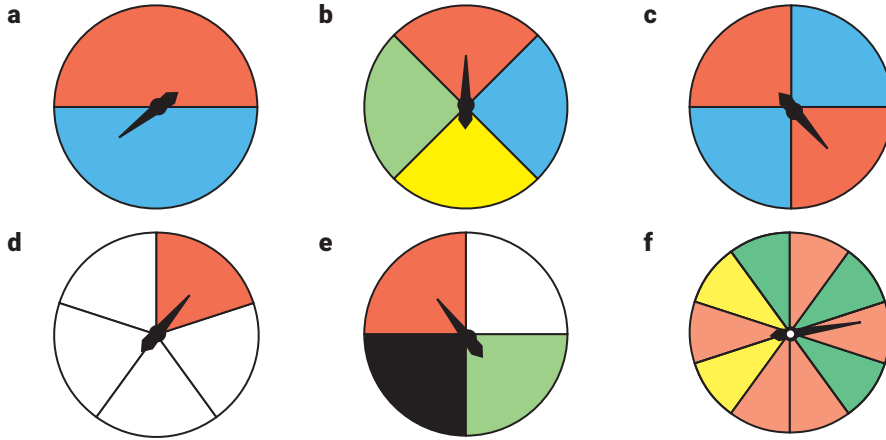
'At least 4' means the smallest number is 4; that is, 4 or more.

Probability **UFRC**

EXAMPLE 1

1 For each spinner:

- i write down the sample space **c**
- ii for one spin, calculate the probability that the spinner lands on red.



2 For each experiment, count the number of possible outcomes and state whether each outcome is equally likely. **c**

- a tossing a coin
- b the result of a rugby match when Australia plays New Zealand
- c the first letter of a person's name
- d the gender of a baby
- e the last digit of a car number plate
- f the result of a driving test

3 List the outcomes in each event. **c**

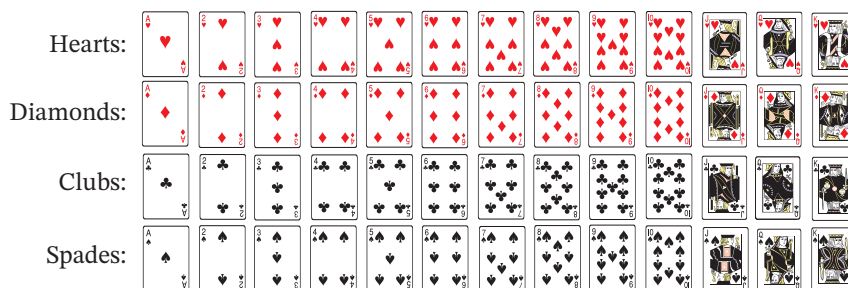
- a rolling an odd number on a die
- b selecting a vowel from the letters of the alphabet
- c having a house number greater than 4 but less than 10
- d having a birthday in a month beginning with A
- e living in a state of Australia
- f being in a high school grade

4 A money box contains 4 \$2 coins, 3 \$1 coins, 2 50c coins, 6 20c coins and 5 10c coins. It is shaken and one coin falls out at random. Calculate:

- a $P(50c \text{ coin})$
- b $P(\$1 \text{ coin})$ (answer as a decimal)
- c $P(10c \text{ or } 20c \text{ coin})$, as a percentage
- d $P(\text{not a } 10c \text{ coin})$
- e $P(\text{gold coin})$, as a decimal
- f $P(\text{a coin under } \$1)$, as a percentage

EXAMPLE 2

- 12** The 52 cards in a standard deck of playing cards are shown below, divided evenly into 4 suits: hearts, diamonds, clubs, and spades.



The cards are shuffled and one is taken out at random.

- How many outcomes are in the sample space?
 - Is each outcome equally likely?
 - How many Aces are in the deck?
 - How many hearts cards are there?
 - How many red 7s are there?
 - Find the probability of selecting:
 - a red card
 - a clubs card
 - the King of spades
 - a Queen
 - a card with an even number
 - a black picture card
- 13** The weather forecaster says that the probability of a rainy day in April is 30%.
- Is this a high probability or a low probability?
 - About how many rainy days are expected in April?
- 14** Your maths teacher calls out a name randomly from your class roll. What is the probability that it is:
- your name?
 - a girl's name?
 - someone aged 14?
 - someone with brown hair?
- 15** An Esky contains 8 cans of lemonade, 5 cans of orange drink and 2 cans of lime drink. How many cans of cola must be added so that the probability of randomly selecting: **R**
- a lemonade is 50%?
 - a lime drink is $\frac{1}{12}$?
 - an orange drink is 0.25?
 - a cola can is $\frac{2}{5}$?



Complementary events

In everyday life, we use the word 'complementary' to describe things that go together and 'complete the picture' when they are together. For example, when dressing for an occasion:

- a shirt and a matching tie complement each other
- a dress and a matching pair of shoes complement each other

Remember also that 'complementary' angles add to 90° .

In probability, **complementary events** are events that together make up all the possible outcomes.

Event	Complementary event
Tossing a tail on a coin	Tossing a head on a coin
Rolling a 6 on a die	Rolling any of the other numbers, from 1 to 5, on a die
Raining	Not raining
Being born on a Monday	Being born on a day other than Monday

- Suppose that there is an equal chance of being born on any day of the week: Monday to Sunday.
 - What is $P(\text{Tues})$, the probability of being born on a Tuesday?
 - What is $P(\text{not Tues})$, the probability of being born on a day other than Tuesday?
 - What do you notice about $P(\text{Tues}) + P(\text{not Tues})$?
- A fruit bowl contains 7 apples, 4 oranges and 9 bananas. One piece of fruit is selected at random from the bowl.
 - Find $P(\text{orange})$.
 - Find $P(\text{not orange})$.
 - What do you notice about $P(\text{orange}) + P(\text{not orange})$?
- A baby is selected at random from the maternity section of a large hospital. There is an equal chance of the baby being a boy or a girl.
 - Find $P(\text{boy})$.
 - Find $P(\text{girl})$.
 - What do you notice about $P(\text{boy}) + P(\text{girl})$?
- Copy and complete the following sentence.
 The probability of an event _____ the probability of its complementary event must always equal _____.

Did you know?



Two-up

Two-up is an Australian coin-tossing game. The game was very popular with soldiers during World War I and it is only legal to play it on Anzac Day (25 April). Two-up involves the tossing of 2 coins and betting on 'heads' or 'tails' on both coins.

Heads: both coins land heads up.

Tails: both coins land tails up.

Odds: one head and one tail.

There is no winner if 'odds' are thrown and the coins are tossed again.

Write down the sample space for this game.

What is the probability of throwing 'odds'?



AAP Photo/AP/Rick Rycroft

9.01

Complementary events

9.02

In any situation, the probabilities of all possible outcomes must add to 1.

Complementary events are 2 events that together make up all the possible outcomes, such as a **head** and a **tail** when tossing a coin. The **complement** of an event E , are all those outcomes that are *not* E , or that are the 'opposite' of E .

Because an event and its complement cover all possible outcomes, the sum of their probabilities must equal 1.



Complementary events



Theoretical probabilities

Complementary events

$$P(E) + P(\text{not } E) = 1$$

$$\text{or } P(\text{not } E) = 1 - P(E)$$

$$\text{or } P(\text{complementary event}) = 1 - P(\text{event})$$

$$\text{or } P(\text{event not occurring}) = 1 - P(\text{event occurring})$$

If the probability is written in percentage form, then $P(\text{not } E) = 100\% - P(E)$

Example 3

A pack of 20 cards contains 10 red, 6 yellow and 4 green cards. One card is drawn from the pack at random. Find the probability that this card is:

- a yellow
- b not yellow
- c not green



Complementary events

Solution

a $P(\text{yellow}) = \frac{6}{20} = \frac{3}{10}$

b $P(\text{not yellow}) = 1 - P(\text{yellow})$
 $= 1 - \frac{3}{10}$
 $= \frac{7}{10}$

The complement of 'yellow' is 'not yellow', which is 'red or green'

Note that $P(\text{yellow}) + P(\text{not yellow}) = \frac{3}{10} + \frac{7}{10} = 1$, which covers all possible outcomes.

c $P(\text{green}) = \frac{4}{20} = \frac{1}{5}$

The complement of 'green' is 'not green', which is 'red or white'

$P(\text{not green}) = 1 - P(\text{green})$
 $= 1 - \frac{1}{5}$
 $= \frac{4}{5}$

Note that $P(\text{green}) + P(\text{not green}) = \frac{1}{5} + \frac{4}{5} = 1$, which covers all possible outcomes.

EXERCISE 9.02 ANSWERS ON P. 572

Complementary events **U F P S R C**

1 Write the complement of each event. **C**

- | | |
|---|-----------------------------------|
| a Tossing tails on a coin | b Cloudy day tomorrow |
| c Selecting a white chocolate from a box containing white and brown chocolates | |
| d Rolling a 6 on a die | e Winning a game of hockey |
| f Selecting a hearts card from a deck of cards | |
| g Being born in winter | h Being under 15 years old |

2 A pile of 11 cards is placed on a desk. There are 6 red cards and the rest are blue. If one card is selected at random, what is the probability that it is not red? Select the correct answer **A**, **B**, **C** or **D**.

A $\frac{5}{11}$

B $\frac{6}{11}$

C $\frac{6}{5}$

D $\frac{5}{6}$

3 A die is rolled. What is the probability that the result is:

- | | | |
|---------------------|----------------------|---------------------|
| a a 3? | b not a 3? | c not even? |
| d not prime? | e at least 2? | f at most 4? |

4 There are 6 books on a shelf: 3 Mathematics, 2 History and 1 Sport. If one book is chosen at random, what is the probability of selecting:

- | | |
|------------------------------------|--------------------------------------|
| a a History book? | b a book that is not History? |
| c a book that is not Sport? | d a book that is not Science? |

EXAMPLE
3

- 5** A car park has 450 cars, 100 motor bikes and 10 buses. One of them is selected at random.
- What is the probability that it is a car?
 - What is the probability that it is not a car? Select **A**, **B**, **C** or **D**.
A $\frac{45}{56}$ **B** $\frac{110}{450}$ **C** $\frac{10}{560}$ **D** $\frac{11}{56}$
- 6** What is the probability that your maths teacher was born in a month:
- beginning with the letter M?
 - that does not begin with the letter M?
- 7** What is the decimal probability that a mobile phone number selected at random does not end in 0 or 1?
- 8** Tahnee buys 5 tickets in a raffle in which 1000 tickets are sold and there is only one prize. What is the probability of Tahnee not winning the prize?
- 9** A jar contains 40 red, 25 blue, 50 black and 15 white jelly beans. One jelly bean is selected at random. What is the probability that it is:
- white?
 - not white?
 - not yellow?
 - not blue or black?
 - not red?
 - not black or white?
- 10** Write the probability of the event that is **complementary** to each event. **C**
- The probability of choosing a Jack from a pack of cards is $\frac{1}{13}$.
 - The chance of shooting a basketball hoop is 61%.
 - The probability of winning a prize is 0.15.
- 11** In a bag of toy cars there are only 3 colours: red, blue and white. If you take out a car at random, the chance of it being red is 0.4, and the chance of it being white is 0.25. **R**
- What is the chance of selecting 'red or white'?
 - What is the chance of the car you select being blue?
 - If the bag holds 40 cars, how many of each colour would you expect to find?
 - What is the chance of the car you select being pink?
- 12** 4 students, Sue, Liam, Emily and Matt, write their names on cards and place the cards in a bag. A card is chosen, without looking, to select the class captain.
- Find the probability that Emily was not chosen.
 - Find the probability that the captain is a boy.
 - What is the chance that the captain is not a boy?
 - What is the chance that the captain is the teacher?
- 13** The probability that it will rain this weekend is 85%. What is the probability that it won't rain?



14 Which of the following is the complementary event to ‘winning a race’?

Select **A**, **B**, **C** or **D**. **R C**

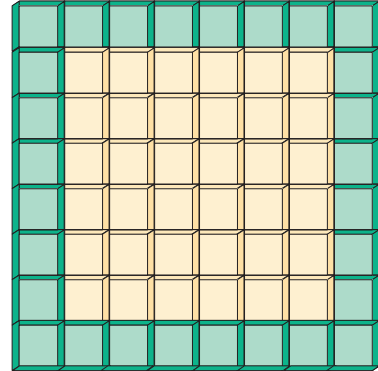
A coming last

B coming second or third

C not winning the race

D coming second

15 A game involves throwing a coin into a grid of squares. For a player to win, the coin must land in a yellow square. If the coin lands outside the grid or between squares, the throw is not counted and the player has another try.



For each throw, what is the probability of:

a not winning?

b winning?

16 The letters of the word PROBABILITY are written on separate cards and one is selected randomly. What is the probability that a letter drawn out is:

a not P?

b not a vowel?

c not I?

d not A or B?

17 In a family there are Mum and Dad, 2 daughters and a son. Each day, they take it in turns to wash the dishes after dinner. If you visit the family, what is the chance that the person washing up is:

a male?

b not a parent?

c not Mum?

d not male?

18 A random 4-digit number is to be formed from the digits 1, 3, 5 and 8. What is the probability that the number will: **PS R**

a start with the digit 5?

b be an odd number?

c be greater than 5000?

9.03 Venn diagrams



Venn diagrams



Venn diagrams group clues



Venn diagrams



Venn diagrams matching activity

A **Venn diagram** is a diagram that uses circles (usually overlapping) to group items into categories. A rectangle represents the whole group while the circles represent categories. Items common to 2 or more categories are placed in the intersection (overlapping region) of the circles. The Venn diagram was invented in 1880 by English mathematician and priest, John Venn (1834–1923).



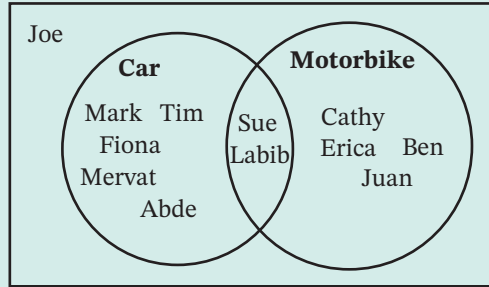
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Foundation Standard Complex

Example 4

A group of people were surveyed on the type of vehicle they drove, and the results are shown on the Venn diagram.

- a How many people were surveyed?
- b How many people drive a car?
- c How many people drive a car or ride a motorbike?
- d Who drives a car and rides a motorbike?
- e Who only rides a motorbike?
- f Why is Joe outside of the circles?



Solution

- a 12 people were surveyed.
- b 7 people drive a car.
- c 11 people drive a car or ride a motorbike.
- d Sue and Labib can drive a car and ride a bike.
- e Cathy, Erica, Ben and Juan can only ride a bike.
- f Joe does not drive a car or ride a bike.

The people within the Car circle.

All the people within the circles.

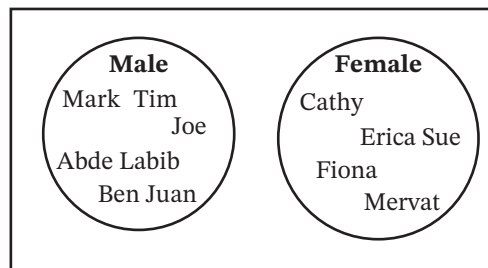
The people in the intersection of the circles.

Mutually exclusive vs overlapping events

Sometimes, 2 categories must be represented on a Venn diagram as 2 separate circles because it is not possible for them to overlap. Here is an example:

In this case, it is not possible to be male *and* female. This is an example of a **mutually exclusive event**. Mutual means 'shared feature' and exclusive means 'belonging to one group only' (such as an exclusive party or an exclusive news story).

However, most Venn diagrams (as in Examples 4 and 5) describe **overlapping events**, or **non-mutually exclusive events**.



'And' vs 'or'

For 2 categories or events A and B, the phrase '**A and B**' means to have both of them occurring together. For example, 'to drive a car' **and** 'to ride a motorbike' in Example 4 means to do both things.

If A and B are **overlapping**, the phrase '**A or B**' means to have A or B or both. For example, 'to drive a car' **or** 'to ride a motorbike' in Example 4 means to drive a car only, or to ride a motorbike only, or to do both. In this case, 'A or B' actually **includes** 'A and B' so this is an example of an **inclusive** 'or'.

If A and B are **mutually exclusive**, the phrase '**A or B**' means to have A only or B only (but not both). For example, 'male' **or** 'female' means to be male, or female, but not both. In this case, 'A or B' **excludes** 'A and B' so this is an example of an **exclusive** 'or'.



Venn diagrams



Venn diagrams 1



Venn diagrams 2

Example 5

30 students in a class were surveyed on how they relaxed after school.

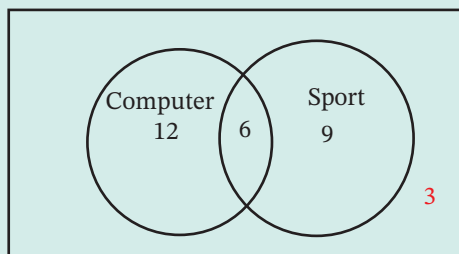
- 18 play on their computer
 - 15 play sport
 - 6 play sports and play on their computer
- a** Show this information on a Venn diagram.
- b** If one student is selected at random from this class, find the probability that the student:
- i** plays sport, but not on their computer
 - ii** plays sport or on their computer, but not both



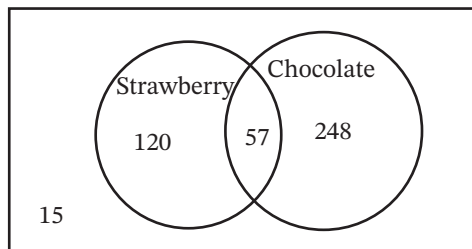
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Solution

- a** When drawing Venn diagrams always begin with the intersection.
- 6 students belong to both groups
 - 18 play on the computer but 6 have already been counted, so $18 - 6 = 12$ play on the computer only
 - 15 play sport but 6 have already been counted, so $15 - 6 = 9$ play sport only
 - $12 + 6 + 9 = 27$, but there are 30 students in the class, which means that 3 students do not belong to either group
- b i** $P(\text{plays sport but not computer}) = \frac{9}{30} = \frac{3}{10}$
- ii** $P(\text{plays sport or computer but not both}) = \frac{12+9}{30} = \frac{21}{30} = \frac{7}{10}$



- 4** A survey was carried out by an ice-cream shop to decide whether chocolate or strawberry was the more popular flavour. **R C**



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- How many people were surveyed?
- How many people liked strawberry or chocolate, but not both?
- How many people liked neither strawberry nor chocolate?
- If a person is randomly chosen from the survey, what is the probability that the person likes both chocolate and strawberry? Select **A**, **B**, **C** or **D**.

A $\frac{57}{440}$

B $\frac{368}{440}$

C $\frac{425}{440}$

D $\frac{57}{425}$

EXAMPLE
5

- 5** The Tourism Council surveyed 130 people to find whether they prefer New South Wales or Queensland as a holiday destination. **R C**

NSW 44

Queensland 66

NSW and Queensland 20

- Construct a Venn diagram to represent the results.
- How many people like NSW or Queensland?
- How many people like NSW or Queensland, but not both?
- What is the probability that a person likes NSW exclusively?
- What is the probability that a person does not like NSW or Queensland?

- 6** **a** Draw a Venn diagram representing the eye colours of this group of students. **R C**

Blue eyes 32

Brown eyes 38

Neither blue nor brown eyes 10

- Are these groups mutually exclusive or not? Why?
- How many students are in the group?
- How many students have blue or brown eyes?
- How many students have blue and brown eyes?
- What is the percentage probability that a student chosen at random from this group does not have brown eyes?

- 7** A retailer surveyed the first 30 customers to see what they bought. **PS R C**

20 bought milk

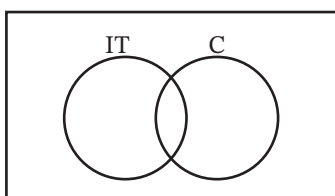
13 bought bread

1 bought neither milk nor bread

- Show this information on a Venn diagram.
- How many customers bought milk and bread?
- How many customers bought milk only?
- What is the probability that a customer randomly chosen:
 - bought bread only?
 - did not buy milk?
 - bought milk or bread?
 - bought milk or bread, but not both?

- 8** There is a class of 30 students, all of whom study Information Technology or Chemistry. 7 of them study both subjects, and 20 study Information Technology. **PS R C**

- a** Copy and complete the Venn diagram to find how many study Chemistry.



- b** If one student is chosen at random, what is the probability that the student studies Chemistry?

Two-way tables

9.04

A **two-way table** is another way of grouping items into overlapping categories, especially when there are many overlaps that cannot be represented by Venn diagrams easily.

Example 6

50 students were surveyed on whether they preferred dogs or cats more as pets. The results were sorted into this two-way table.

- How many students do not like dogs or cats?
- How many students like cats?
- How many boys like cats?
- How many students:
 - like cats or are boys?
 - like cats or are boys, but not both?
- What is the probability that a student selected at random from the survey likes dogs?

	Preferred pet	
	Dog	Cat
Boys	12	4
Girls	11	15



Two-way tables



Two-way tables



Two-way probability tables



Two-way probability tables

Solution

a $12 + 4 + 11 + 15 = 42$

Students who do not like dogs or cats = $50 - 42$
 $= 8$

b Students who like cats = $4 + 15$
 $= 19$

c Boys who like cats = 4

d i Students who like cats or who are boys = $12 + 4 + 15$
 $= 31$

ii Students who like cats or who are boys but not both = $12 + 15$
 $= 27$

e Students who like dogs = $12 + 11$
 $= 23$

Probability of selecting a student who likes dogs = $\frac{23}{50}$

Adding the values in the table.

50 students in survey.

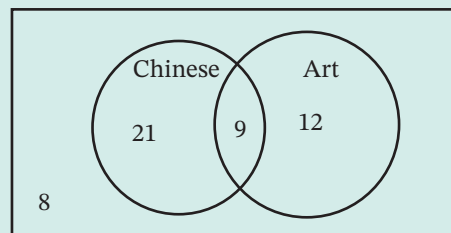


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Example 7

A group of 50 Year 9 students were grouped in a Venn diagram according to whether they took Chinese or Art as an elective subject.

Represent this information in a two-way table.



Solution

From the Venn diagram:

- 9 students take Chinese and Art
- 21 students take Chinese, but not Art
- 12 students take Art, but not Chinese
- 8 students do not take Chinese or Art

	Chinese	Not Chinese	
Art	9	12	21
Not Art	21	8	29
	30	20	50

Two-way tables **U F R C**

EXAMPLE
6

- 1** A primary school class was surveyed on whether its students could swim. The results are shown below. **R C**

	Can swim	Cannot swim
Boys	13	2
Girls	9	3

- How many students are in the class?
- How many students are boys or cannot swim?
- How many students are boys and cannot swim?
- What is the probability that a student randomly selected from this class is a girl?
- What is the probability that a student selected at random is:
 - a non-swimmer?
 - a girl who can swim?

- 2** The players of a soccer club were divided into groups according to their age and weight. **R C**

	Heavy	Light
Junior	64	96
Senior	144	32

- How many players does the club have?
- How many players are juniors or light?
- How many players are juniors or light, but not both?
- What is the probability that a player selected at random is:
 - a senior?
 - a junior and heavy?
 - a senior or light?

- 3** This incomplete table describes the audience watching a movie at a cinema. **R C**

	Under 18	Over 18	Total
Female		142	198
Male	45		
Total			344

- Copy and complete the table.
- How many males were in the audience?
- How many under 18 females were there?
- If a person is selected at random from the audience, what is the probability that the person:
 - is male and over 18?
 - is male or over 18?
 - is male or over 18, but not both?
 - is over 18?

- 4** A group of children were asked whether they liked carrots. **R C**

	Likes carrots	Dislikes carrots	Total
Boys	75		200
Girls		40	
Total	145		



Shutterstock.com/Shanta Giddens

- Copy and complete the table.
- How many children dislike carrots?

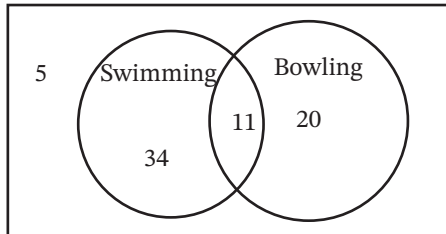


- c** What is the probability that a child randomly selected is:
- i** a girl? **ii** a boy or dislikes carrots? **iii** a boy and dislikes carrots?
- d** What is the chance that a child randomly selected is a girl and likes carrots?
Select **A**, **B**, **C** or **D**.

- A** $\frac{29}{62}$ **B** $\frac{11}{31}$ **C** 1 **D** $\frac{7}{31}$

EXAMPLE
7

- 5** The Venn diagram below shows the number of students who participate in swimming or bowling regularly. Copy and complete the two-way table for this data. **R C**

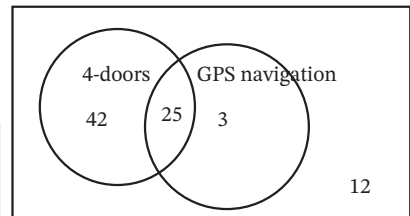


	Swimming	Not swimming	Total
Bowling			
Not bowling			
Total			

- 6** The Venn diagram compares 2 features of the cars on display at Rusty Motors. **R C**

- a** Copy and complete the two-way table.

	GPS navigation	No GPS navigation	Total
4-doors			
Not 4-doors			
Total			



- b** A car is chosen at random from the car yard. What is the probability that it has:
- i** GPS navigation? **ii** 4 doors?
- iii** 4 doors and GPS navigation? **iv** no GPS navigation?

- 7** A group of university students were tested to see if they needed glasses. The table shows the results. **R C**

	Needs glasses	Does not need glasses	Total
Male	22		82
Female		85	98
Total		145	

- a** Copy and complete the table.
- b** If a student is chosen randomly from this group, what is the probability that the student:
- i** is female? **ii** needs glasses?
- iii** is female and needs glasses? **iv** is male or does not need glasses?
- v** is female or needs glasses? **vi** is male or does not need glasses, but not both?

Technology

Rolling a die

In this activity, you need a scientific calculator or spreadsheet to simulate the rolling of a die.

- 1 Copy this table.

Outcome	Frequency
1	
2	
3	
4	
5	
6	



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- 2 Random numbers from 1 to 6 can be generated on a calculator or spreadsheet using the RanInt, RANDOM or =RAND() functions.

Casio scientific calculator

Enter the formula RanInt#(1,6) as shown:

(ALPHA) **·** **1** **SHIFT** **)** **6** **)** **=**

Sharp scientific calculator

(2nd F) **7** (RANDOM), select 1 R-DICE, **=**

Press **=** more times to generate more random integers from 1 to 6.

On a spreadsheet, type **=INT(RAND()*6+1)** into a cell and press the Enter key, then the **F9** key repeatedly for more random numbers.

- 3 Repeat the simulation 20 times and record the results in your table.
- 4 Are certain numbers more likely to be rolled than others? For example, is a 2 more likely to be rolled than a 5? Do your results reflect this?
- 5 Compare your results with the simulated results of other members in your class. Are they similar or different? Are they what you and your classmates expected? Discuss.

Multiplying or dividing by a multiple of 10

1 Study each example.

a $4 \times 700 = 4 \times 7 \times 100 = 28 \times 100 = 2800$

b $5 \times 60 = 5 \times 6 \times 10 = 30 \times 10 = 300$

c $12 \times 40 = 12 \times 4 \times 10 = 48 \times 10 = 480$

d $3.2 \times 30 = 3.2 \times 3 \times 10 = 9.6 \times 10 = 96$ (by estimation, $3 \times 30 = 90 \approx 96$)

e $4.5 \times 50 = 4.5 \times 5 \times 10 = 22.5 \times 10 = 225$ (by estimation, $5 \times 50 = 250 \approx 225$)

f $9.4 \times 200 = 9.4 \times 2 \times 100 = 18.8 \times 100 = 1880$
(by estimation, $9 \times 200 = 1800 \approx 1880$)

2 Now evaluate each product.

a 8×2000

b 3×70

c 11×900

d 2×300

e 4×4000

f 5×80

g 7×70

h 1.3×40

i 2.5×600

j 6.8×200

k 3.9×50

l 4.4×4000

3 Study each example.

a $8000 \div 400 = 80\cancel{00} \div 4\cancel{00} = 80 \div 4 = 20$

b $200 \div 50 = 20\cancel{0} \div 5\cancel{0} = 20 \div 5 = 4$

c $6000 \div 20 = 600\cancel{0} \div 2\cancel{0} = 600 \div 2 = 300$

d $282 \div 30 = 282 \div 10 \div 3 = 28.2 \div 3 = 9.4$

e $3520 \div 40 = 352\cancel{0} \div 4\cancel{0} = 352 \div 4 = 88$

f $8940 \div 200 = 8940 \div 100 \div 2 = 89.4 \div 2 = 44.7$

4 Now evaluate each quotient.

a $560 \div 70$

b $2500 \div 50$

c $3200 \div 400$

d $440 \div 20$

e $160 \div 40$

f $1500 \div 30$

g $450 \div 50$

h $744 \div 80$

i $2550 \div 300$

j $846 \div 200$

k $576 \div 60$

l $2040 \div 50$

Probability problems

9.05

The following exercise involves all the probability ideas that have been covered so far.

EXERCISE 9.05 ANSWERS ON P. 573

Probability problems **U F P S R C**

- 1** There were 28 students on the school trip to Japan: 17 were born in Australia, 3 in Italy, 4 in Vietnam, 1 in Japan and 3 in China. One student was chosen at random. Find the probability that the student was:
- a** born in Australia **b** born in Asia **c** not born in Vietnam.
- 2** The 7 letters of the word SUCCESS are written on separate cards. A card is selected at random and the letter noted. **R C**
- a** List the sample space.
b Is each letter equally likely to be selected? Explain.
c Find the chance that the letter chosen:
- i** is an S **ii** is a vowel
iii is not a C **iv** is also a letter of the word FAIL.
- 3** Alex selects one sock at random from a bag containing 2 black, 2 blue and 2 red socks. **R C**
- a** List the sample space.
b Find the probability that Alex selects:
- i** a blue sock **ii** a black sock **iii** a pink sock
c What is the complementary event to choosing a red sock? What is the probability of this event?
- 4** A spinner is evenly divided into 5 sections numbered 1, 2, 3, 4 and 5. For one spin, find the probability, as a percentage, that it lands on:
- a** 2 **b** an even number
c a number less than 5 **d** a number at most 5
- 5** Sophia bought 4 tickets in a lottery in which there were 100 000 tickets. What is the probability that she won first prize? Select the correct answer **A, B, C** or **D**.
- A** 0.000 0025 **B** 0.000 25 **C** 0.04 **D** 0.000 04
- 6** One ball is selected at random from a barrel of balls numbered 1 to 100. Find the probability that the number shown on the ball is:
- a** 12 **b** greater than 40 **c** even **d** at most 85
- 7** Stathis flipped a coin 7 times and a tail showed each time. What is the chance of a tail showing on the next toss? **R**



Matching probabilities

9.05

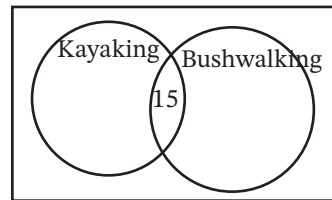
8 A computer selected a random number from 1 to 15 inclusive. Find the probability that the number is:

- a** odd
- b** not prime
- c** a multiple of 3 or 5
- d** a factor of 12

9 In a tennis tournament there are 60 players. Of these, 34 are from NSW, 20 are from Victoria and 6 are from Queensland. Find the probability that a player selected at random from the tournament is: **R C**

- a** from NSW
- b** from Queensland
- c** not from Victoria
- d** from Queensland or Victoria.

10 40 students at a school camp can select kayaking or bushwalking as an activity, or both. A total of 35 students chose kayaking, 20 chose bushwalking, including some who chose both. All students chose an activity. **PS R C**



- a** Copy and complete the Venn diagram representing these students.
- b** What is the probability that a student, chosen at random:
 - i** chose both activities?
 - ii** did not choose bushwalking?

11 The probability that a carton of eggs contains any broken eggs is 0.1. Find the probability that a carton contains no broken eggs.

12 A sample of students from a sports high school were surveyed on whether they participated in hockey or judo. The results are shown in this two-way table. **R C**

	Judo	Not Judo	Total
Hockey	128		360
Not hockey	100	40	
Total			

- a** Copy and complete the table.
- b** What is the probability that a student chosen randomly:
 - i** does judo?
 - ii** doesn't do hockey?
 - iii** does judo, but not hockey?
 - iv** does hockey and judo?
 - v** does hockey or judo?
 - vi** does hockey or judo, but not both?

13 In a bag there are lollies of 3 different colours: red, green and purple. The chance of taking out a red lolly at random is 0.5, the chance of it being green is 0.2. **PS R C**

- a** What is the chance of the lolly being purple?
- b** If the bag holds 10 lollies, how many of each colour would you expect to find?
- c** If a second bag had 100 lollies, how many of each would you expect to find?
- d** Which bag would give you a better chance of picking a red lolly? Explain your answer.

Relative frequency

9.06

Probability calculated using the formula:

$$P(E) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$$

is more specifically called **theoretical probability**.

We can also determine the probability of an event based on the results of an experiment or trial that has been repeated many times, such as the safety testing of different cars, or rely on past statistics, such as the number of rainy days in April. This type of probability is called **experimental probability** or **observed probability**, which is based on **relative frequency**, the number of times an event occurred as a fraction of the total frequency of outcomes.

Experimental probability

$$P(E) = \frac{\text{number of times the event happened}}{\text{total number of trials}}$$

or $P(E) = \frac{\text{frequency of } E}{\text{total frequency}}$

As in statistics, 'frequency' means the number of times something happens.

Expected frequency is the expected number of times an event will occur over repeated trials.

Expected frequency

$$\text{Expected frequency} = \text{theoretical probability} \times \text{number of trials}$$

Example 8

Declan rolled a die 60 times and recorded the results in a table.

Outcome	1	2	3	4	5	6
Frequency	10	11	13	7	8	11

- What is the theoretical probability of rolling a 5 or 6 on a die?
- For 60 rolls, what is the expected frequency of rolling a 5 or 6? How does this compare with the observed (actual) frequency?
- What is the experimental probability of rolling a 5 or 6?

Solution

a $P(5 \text{ or } 6) = \frac{2}{6} = \frac{1}{3}$

b Expected frequency of 5s or 6s $= \frac{1}{3} \times 60$ Probability \times number of trials
 $= 20$

From the table, the observed frequency of 5s and 6s $= 8 + 11 = 19$, which is close to the expected frequency, 20.

c Experimental $P(5 \text{ or } 6) = \frac{8+11}{60} = \frac{19}{60}$



Relative frequencies



Dice probability



Spinning chance



A page of spinners



Experimental probabilities



Experimental probability



Duelling dice



Experimental probability



Theoretical probability



Probability review

EXAMPLE
8

Relative frequency **UFRC**

1 Harper rolled a biased die 60 times and the number 2 came up 15 times.

- a** What is the experimental probability of rolling a 2?
- b** If the same die was rolled 500 times, what is the expected frequency of rolling a 2?

2 a Copy this table.

b Flip a coin 50 times and record the results in the table.

Outcome	Tally	Frequency
Head		
Tail		
Total		

c Find, as a decimal, the experimental probability of flipping:

- i** a head
- ii** a tail

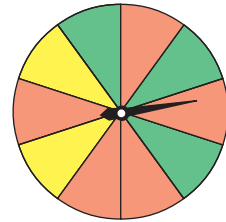
d Find, as a decimal, the theoretical probability of flipping:

- i** a head
- ii** a tail

e How do the experimental probabilities compare with the theoretical probabilities? **c**

3 Vinh spun this spinner 80 times and found the following results:

Outcome	Red	Green	Yellow
Frequency	44	25	11



- a** What is the theoretical probability of spinning red?
- b** For 80 spins, what is the expected frequency of spinning red? How does this compare with the observed (actual) frequency? **c**
- c** What is the experimental probability of spinning red?
- d** What is the theoretical probability of spinning yellow?
- e** Find the expected frequency of spinning yellow over 80 spins and compare this with the observed frequency. **c**
- f** What is the experimental probability of spinning yellow?

4 The matches in a sample of matchboxes were counted. The number of matches in each box are recorded below. **R**

Number of matches	48	49	50	51	52	53
Number of matchboxes	3	6	20	16	4	1

- a** How many matchboxes were tested?
- b** What is the experimental probability of finding 51 matches in a box?
- c** What is the experimental probability of finding fewer than 50 matches in a box?
- d** In 1000 matchboxes, how many matchboxes would you expect to contain:
 - i** exactly 50 matches?
 - ii** at least 50 matches?



5 a Copy this table. **R C**

Outcome	Tally	Frequency
1		
2		
3		
4		
5		
6		
	Total	

b Roll a die 72 times and record the results in the table.

c Find the theoretical probability of rolling:

- i** 4
- ii** a number greater than 2
- iii** an even number

d When rolling a die 72 times, what is the expected frequency of rolling:

- i** 4?
- ii** a number greater than 2?
- iii** an even number?

e How do your expected frequencies compare with the observed frequencies from the table?

f Find the experimental probability of rolling:

- i** 4
- ii** a number greater than 2
- iii** an even number

g If a die was rolled 1000 times, how many times should 4 come up:

- i** based on the theoretical probability?
- ii** based on the experimental probability?

6 Sefina tossed a coin many times and got 135 heads and 115 tails. Calculate, as a percentage, the experimental probability of tails with this coin.

7 A die was rolled 80 times, with the results shown below. **R C**

Outcome	1	2	3	4	5	6
Frequency	11	13	9	13	12	22

- a** Is each outcome equally likely?
- b** Do you think this die is biased (unfair)? Give a reason for your answer.
- c** Write, as a percentage, the experimental probability of rolling a 1 on this die.
- d** If this die was rolled 100 times, what is the expected frequency of 3 coming up?

- 8** A pair of dice was rolled 50 times and their sum calculated each time.

Sum	2	3	4	5	6	7	8	9	10	11	12
Frequency	0	2	4	6	5	5	9	6	8	3	2

- a** Find, as a decimal, the experimental probability of rolling a sum:
- i** of 11
 - ii** of 5 or 6
 - iii** that is odd
 - iv** of 6 or more
 - v** of at most 6
 - vi** that is a composite number
- b** Which sum(s):
- i** was most likely?
 - ii** had a probability of $\frac{3}{25}$?
 - iii** was least likely?
 - iv** had a probability of 16%?
 - v** was second-most likely?
 - vi** had a probability of $\frac{1}{10}$?



Coin toss experiment



Coin tossing simulator



Die rolling simulator



Simulating a spinner



Spinner simulator

Technology

Tossing a coin

In this activity, a spreadsheet is used to simulate the tossing of a coin. The computer can quickly generate either a 1 (representing heads) or a 2 (tails). We will use the command **RAND** to generate these random numbers.

- In cell A1 enter the label 'Tossing a coin'.
- In cell A2, enter the formula **=INT(RAND()*2+1)**. Press Enter and either 1 or 2 should appear randomly in the cell.
- Select cell A2 and **Fill Down** to cell A11 to generate random numbers (1 or 2), to simulate the tossing of a coin 10 times.
- Your results will probably be different from those of other students in the class. Compare.
- Copy the table below and in the first blank row record your results for Trials 1–10 (the numbers of heads and tails in your first 10 'tosses').

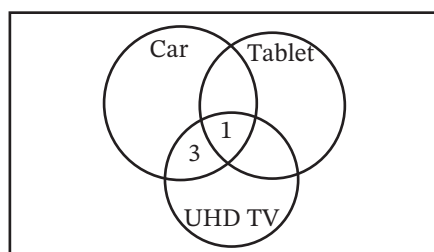
Trial	Number of heads	Number of tails
1–10		
11–20		
21–30		
31–40		
41–50		
51–60		
61–70		
71–80		
81–90		
91–100		
Total		

- 6 Simulate another 10 tosses of the coin by making the spreadsheet generate another set of random numbers. Do this by placing the cursor between the tops of columns A and B (so that it turns into a 'double arrow') and clicking.
- 7 Enter the results for Trials 11–20 in your table.
- 8 Repeat 8 more times so that you have 100 tosses of the coin recorded in the table.
- 9 **a** For 100 tosses of a coin, what is the expected frequency of heads based on the theoretical probability?
 - b** Compare this with the observed frequency.
- 10 Calculate, as a decimal, the theoretical and experimental probabilities of tossing heads.
- 11 Compare your results with those of other students in your class. Briefly explain any differences and why they may have occurred.
- 12 Combine the results of students in your class to calculate the experimental probability of tossing heads, as a decimal. How does this compare with the theoretical probability?

Power plus ANSWERS ON P. 574

- 1 A market research survey of 50 people shows that 42 people own a car, 20 own a tablet device and 8 own a UHD TV. Of these:
 - 4 own a car and a UHD TV
 - 3 own a UHD TV and a tablet
 - 14 own a car and a tablet
 - 1 person has a car, a tablet and a UHD TV

Copy and complete the following Venn diagram to represent the survey results.





2 2 dice are rolled and the numbers are **multiplied** together to arrive at a score.

a Copy and complete this table to show all possible products.

		1st die					
		×	1	2	3	4	5
2nd die	1	1	2	3			
	2	2	4				
	3	3	6				
	4			12			
	5						
	6						

b How many different products are possible?

c Why isn't each product equally likely?

d Which product is most likely?

e Which product is least likely?

f What is the probability of a product of:

i 6?

ii 20?

iii at least 20?

g Which product has a probability of $\frac{1}{12}$?

3 3 friends decide to play a game with 2 dice. Danielle wins if the sum of the numbers is 3 or 5, and Vanessa wins if the total is 6 or 8. Any other total means that Karla wins. Is the game fair? Explain your answer.

4 A committee of 2 people is to be selected from 2 boys (Paul and Sumeet) and 2 girls (Tash and Nadine).

a List all the committees you can form.

b If you were to choose a committee at random, what is the probability that it would include Tash?

CHAPTER 9 REVIEW

Language of maths

at least	certain	complementary	event
exclusive	expected frequency	experimental probability	improbable
impossible	inclusive	likely	mutually exclusive
observed frequency	outcome	overlapping	probable
random	relative frequency	sample space	theoretical probability
trial	two-way table	unlikely	Venn diagram



- 1 What is the meaning of **sample space**?
- 2 What is the **complementary event** to winning a soccer match?
- 3 What does it mean when a person is selected '**at random**' for a survey?
- 4 Draw a Venn diagram with categories 'Year 7 students' and 'Year 8 students'. Are these categories **mutually exclusive** or not?
- 5 What term means the number of times an event should occur over repeated trials?
- 6 Explain what 'is right-handed or drives a car' means exactly, given that they are **overlapping** events.

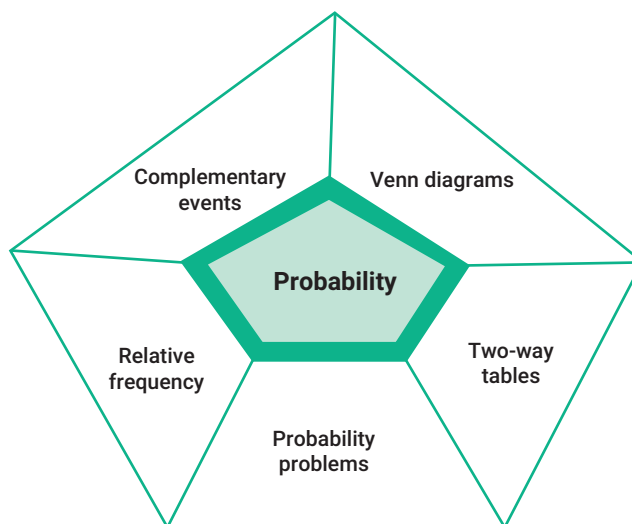
Topic summary

- Write in your own words what you have learnt in this chapter.
- Write which parts of the chapter were new to you.
- Copy and complete:

The things I understand about probability are...

The things I am still not confident in doing in this chapter are...

Print (or copy) and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.



TEST YOURSELF 9 ANSWERS ON P. 574

9.01

1 There are 6 red, 3 yellow and 4 blue hair ties in a drawer. One hair tie is selected at random. What is the probability that it is red? Select the correct answer **A**, **B**, **C**, or **D**.

- A** $\frac{1}{13}$ **B** $\frac{6}{13}$ **C** $\frac{6}{7}$ **D** $\frac{13}{6}$

9.01

2 A die is rolled. Find the probability that the number that comes up is:

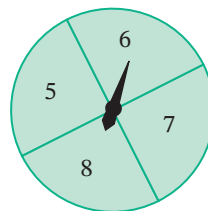
- a** 1 **b** more than 2 **c** odd **d** composite

9.01

3 a Write the sample space for this spinner.

b Find, as a percentage, the probability that the number spun is:

- i** 5 **ii** at least 5
iii 5 or less **iv** less than 5



9.02

4 Write the complement of each event and its probability.

- a** Choosing an Ace from a standard deck of cards
b Rolling a factor of 6 on a die
c Buying the winning ticket out of 1000 tickets sold

9.02

5 A golfer has a probability of 74% of sinking a putt. What is the probability that he will miss a putt?

9.02

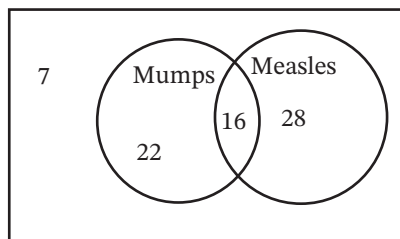
6 A truck carries 240 boxes of hand sanitiser, 305 boxes of soap, 335 boxes of paper and 120 boxes of pens. The driver chooses one box at random. What is the probability that it is:

- a** a box of hand sanitiser? **b** a box of pens or soap?
c a box of paper? **d** not a box of paper?

9.03

7 The medical history of a group of children is shown in the Venn diagram.

- a** How many children are in the group?
b How many children have had the mumps and the measles?
c How many children have had the mumps or the measles, but not both?
d What is the probability that one child selected at random from this group has had:
i the measles?
ii the mumps or the measles?
iii neither the mumps nor the measles?

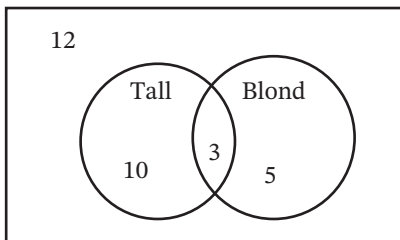


- 8** A group of people were surveyed on whether they watched cricket or soccer on TV. The results were sorted into a two-way table.

	TV sport	
	Cricket	Not cricket
Soccer	28	15
Not soccer	20	7

- How many people were surveyed?
- How many people watch soccer?
- How many people watch soccer or cricket?
- What is the probability that a person selected at random from the survey:
 - watches neither soccer nor cricket?
 - watches soccer or cricket, but not both?
 - does not watch cricket?
 - does not watch soccer?

- 9** Represent the data in the Venn diagram on the two-way table.



	Tall	Not tall
Blond		
Not blond		

- 10** An 8-sided die has 2 red, 3 white, 1 blue and 2 yellow faces. If the die is rolled, find the decimal probability that the face that comes up is:
- blue
 - yellow
 - not red
 - white or yellow
- 11** A die was rolled 80 times and the numbers 1 or 6 came up 25 times.
- What is the experimental probability of rolling 1 or 6, as a percentage?
 - What is the theoretical probability of rolling 1 or 6, as a percentage?
 - Calculate the expected frequency of rolling 1 or 6 over 80 trials.